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METHOD OF CALCULATION OF NONSTATIONARY AERODYNAMIC CHARACTERISTICS OF A LOW-FLYING WING WITH A CYLINDRICAL FUSELAGE

by

L. G. Tsvetkov

FOREIGN TECHNOLOGY DIVISION

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By: L. G. Tsvetkov

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

METHOD OF CALCULATION OF NONSTATIONARY AERODYNAMIC CHARACTERISTICS OF A LOW-FLYING WING WITH A CYLINDRICAL FUSELAGE

L.G. Tsvetkov
Department of Applied and Computer Mathematics

The article gives an account of the method of calculation of nonstationary aerodynamic characteristics of a low-flying wing of complex configuration with a fuselage in the form of an infinite cylinder taking into account the nonlinearity caused by the effect of the setting angle of attack of the wing and the movement of points of the surface of the wing with respect to the wing with nonstationary motion of the complex.

Let us examine the motion of a wing with a fuselage in a nonviscous incompressible fluid with a constant speed u_0 in parallel to the reference surface. Let us assume that the complex accomplishes harmonic vibrations with low relative frequency made up from the translational vertical movement and rotation around the OX and OZ axes of the bound coordinate system OXYZ (see the figure). Vibrations of the complex are accomplished with respect to the average position at which the axis of the fuselage is parallel to the reference surface and is distant from it by the magnitude $h_0 = \frac{h_0'}{b}$, where B is the root chord of the wing.

Let us represent the thin, slightly curved wing of complex configuration in the form of a combination of plates and plans, and let us assign the geometry of the complex by means of $\lambda_к$ - aspect ratio, $\eta_к$ - taper, $\alpha_к$ - setting angle of attack, $\chi_к$ -

sweep angle of the leading edge, ψ_k - deadrise angle of the kth plan of the wing, $\varepsilon = \frac{\varepsilon'}{b}$ - relative rise of the point of intersection of the root chord above the axis of the fuselage and $R = \frac{R'}{b}$ - the relative radius of the fuselage.

Let us present the coefficients of aerodynamic force and moments of the lifting wing by coefficients of rotating derivatives in the form

$$c_y = c_y^0 + \sum_\nu c_y^\nu \nu, \quad m_z = m_z^0 + \sum_\nu m_z^\nu \nu, \quad m_x = m_x^\vartheta \vartheta + m_x^{\dot\vartheta} \dot\vartheta, \qquad \text{(I)}$$

$$\nu = \varphi, h, \vartheta, \varphi, \dot h, \dot\vartheta,$$

where $\varphi = \varphi(t)$ is the small increase in the trim angle of the complex, $h = h(t)$ - increase in the flight altitude, $\vartheta = \vartheta(t)$ - small increase in the bank angle.

The streamline flow around the fuselage, which has the form of an infinite cylinder, vibrating above the screen, is modeled by the system of dipoles with a constant (along the OX axis) intensity with vertical vibrations and with an intensity changing according to the linear law with rotating vibrations with respect to the OZ axis. The position of the dipoles and their intensity are determined by the principle of inversion with respect to the cylindrical surface and the principle of mirror image with respect to the screen, assuming in the first approximation the intensities of the dipoles to be the same as those with vibrations of the cylinder in an infinite medium.[1].

We find the speeds caused by vibrations of the inherent fuselage above the screen at point ξ_j, η_j, ζ_j of the bound coordinate system by the method of successive approximations according to the following equations in which all the linear dimensions belong to the root chord of the wing:

$u_y(t)$ - with translational vertical vibrations with velocity

$$W_{x_j} = 0, \quad W_{y_j} = v^0_{y_j} u_y(t), \quad W_{z_j} = v^0_{z_j} u_y(t), \qquad (2)$$

with rotational vibrations with the angular velocity $\Omega_z(t)$

$$W_{x_j} = \bar{v}^{\circ}_{x_j} b\Omega_z(t), \quad W_{y_j} = \bar{v}^{\circ}_{y_j} b\Omega_z(t), \quad W_{z_j} = \bar{v}^{\circ}_{z_j} b\Omega_z(t), \tag{3}$$

where

$$\bar{v}^{\circ}_{x_j} = -R^2 \sum_{\gamma=1}^{\infty} \sum_{n=1}^{2} \frac{m_n^{\gamma}}{\rho_n^{\gamma}} (\eta_j - \eta_n^{\gamma}), \quad v^{\circ}_{y_j} = -R^2 \sum_{\gamma=1}^{\infty} \sum_{n=1}^{2} \frac{m_n^{\gamma}}{(\rho_n^{\gamma})^2} [\zeta_j^2 - (\eta_j - \eta_n^{\gamma})^2];$$

$$\bar{v}^{\circ}_{y_j} = -R^2 \sum_{\gamma=1}^{\infty} \sum_{n=1}^{2} \frac{m_n^{\gamma}}{(\rho_n^{\gamma})} \xi_j [\zeta_j^2 - (\eta_j^2 - \eta_n^{\gamma})^2], \quad v^{\circ}_{z_j} = 2R^2 \sum_{\gamma=1}^{\infty} \sum_{n=1}^{2} \frac{m_n^{\gamma}}{(\rho_n^{\gamma})} \zeta_j (\eta_j - \eta_n^{\gamma});$$

$$\bar{v}^{\circ}_{z_j} = 2R^2 \sum_{\gamma=1} \sum_{n=1} \frac{m_n^{\gamma}}{(\rho_n^{\gamma})^2} \xi_j \zeta_j (\eta_j - \eta_n^{\gamma}), \quad \rho_n^{\gamma} = (\eta_j - \eta_n^{\gamma})^2 + \zeta_j^2 .$$

The quantities m_n^{γ} and η_n^{γ} entering into the velocity expressions are connected by the dependences

$$m_2^{\gamma} = -m_1^{\gamma}, \quad m_1^{\gamma+1} = \frac{m_1^{\gamma} R^2}{(\eta_2^{\gamma})^2}, \quad \eta_2^{\gamma} = -(2h_0 + \eta_1^{\gamma}), \quad \eta_1^{\gamma+1} = \frac{R^2}{\eta_2^{\gamma}} .$$

In the first approximation ($\gamma = 1$) we must assume that $m_1^1 = 1$, $\eta_1^1 = 0$.

We model the vortex surface of the wing by the system of oblique horseshoe-shaped vortices, and we assume that the free vortices are are first located in the plane of the wing and behind the trailing edge according to the velocity of the non-disturbing advancing flow in parallel to the reference surface. According to recommendations of work [2], we produce the location of the horseshoe-shaped vortices on each plan of the wing and the selection of the check points in which the boundary conditions on the wing are satisfied. The numbering of the check points is continuous for the whole right half of the lift wing; the reading is conducted in each plan, beginning from the first, along the span from the root to the end chord of the plan and along the chord from the leading edge to the trailing edge, where N_k is the number of vortices located along the span of the plan, n_k -

the number of vortices along the chord.

To fulfil the boundary condition of sealing on the reference surface, the vortex system of the wing is mirror reflected with respect to the plane of the screen with opposite signs of the vortices.

The fufilment of the condition of sealing on the cylindrical fuselage with retention of the boundary condition on the reference surface is ensured by the successive inversion of each ith transformed horseshoe-shaped vortex and its image relative to the screen with respect to the cylindrical surface and mirror image relative to the reference surface [3]. The transformed horseshoe-shaped vortex is obtained by the projection of the initial ith vortex of the wing onto the plane which is parallel to the axis of the fuselage and passes through the trailing edge of the plan. As a result of a similar image within the fuselage, there is available a number of horseshoe-shaped vortices of the same intensity as that of the ith vortex of the wing but having different parameters. Replacement of the initial horseshoe-shaped vortex by the transformed vortex with use of the method of successive inversion is admissible in view of the small setting angles of attack of the wing.

Parameters of the inherent ith transformed vortex (l_i - semispan, ψ_i - deadrise angle, χ_i - sweep angle of the connected vortex, ξ_i , η_i and ζ_i - coordinates of the position of the vortex in the bound coordinate system) are calculated in terms of the initial geometric parameters of the complex in the following way:

$$\psi_i = \psi_\kappa , \qquad l_i = \frac{\lambda_\kappa (1+\eta_\kappa)}{4N_\kappa \prod\limits_{p=1} \eta_p} , \qquad \chi_i = \operatorname{arc\,tg} f_i ,$$

$$\xi_i = 0{,}5 - \sum_{p=0}^{\kappa-1} l_p \operatorname{tg} \chi_p - \left\{ 0{,}25 + [\![A_i]\!] \right\} - \mu_i f_i ,$$

$$\eta_i = e + \mu_i \sin\psi_i + \sum_{p=0}^{\kappa-1} l_p \sin\psi_p , \quad \zeta_i = \tau + \mu_i \cos\psi_i + \sum_{p=0}^{\kappa-1} l_p \cos\psi_p ,$$

where

$$\left.\begin{aligned} \mu_i &= \frac{\lambda_k(1+\eta_k)}{4N_k \prod_{p=1}^{k} \eta_p}\left[1 + 2N_k\{\{A_i\}\}\right], \qquad l_k = \frac{\lambda_k(1+\eta_k)}{2\cdot\prod_{p=1}^{k}\eta_p}, \\ f_i &= \operatorname{tg}\chi_k + \frac{2(1-\eta_k)}{\lambda_k n_k(1+\eta_k)}\left[1 + [\![A_i]\!]\right], \qquad A_i = \frac{1}{N_k}\left(i - 1 - \sum_{p=0}^{k-1} n_p N_p\right), \\ \tau &= \begin{cases} \sqrt{R^2 - e^2} & \text{when } |e| < R, \\ 0 & \text{when } |e| \geq R, \end{cases} \end{aligned}\right\} \quad (4)$$

when k = 0 we must assume that $l_k = 0$ and $N_k = 0$; $\{\{\ \}\}$ - operation of the separation of the fraction part; $[\![\ ]\!]$ - operation of the separation of the whole part of the number.

The conformity between the number of the vortex i and the number of the panel k, on which this vortex is located, is found from the condition of the fulfilment of the equality

$$\sum_{p=0}^{k-1} n_p N_p < i \leq \sum_{p=0}^{k} n_p N_p .$$

For an account of the change in velocities at the check points j of the surface of the wing owing to movements of the latter with respect to the vortices reflected relative to the screen with nonstationary motion of the complex, the velocities induced by these vortices we expand in Taylor series with respect to the small parameters of the movements $\Delta\eta_{ij}$ and $\Delta\zeta_{ij}$ and let us be limited to terms of the first order of smallness, so that in the case of the constant (in time) circulation of the horseshoe-shaped vortex $(\Gamma_{+i}(t) = u_0 b\Gamma_i = \mathrm{const})$ the value of the velocity is determined by the equation

$$\bar{W}_{ij} = \frac{u_0\Gamma_i}{4\pi}\left(\bar{w}_{ij} + \Delta\eta_{ij}\frac{\partial\bar{w}_{ij}}{\partial\eta} + \Delta\zeta_{ij}\frac{\partial\bar{w}_{ij}}{\partial\zeta}\right). \quad (5)$$

With the harmonic law of the change in the circulation of the vortex with time, the increase in velocities at points of the surface of the wing owing to their small movements has a

higher order of smallness, and the value of velocities is determined by a well-known manner (at values of the Strouhal numbers tending to zero, $q=\frac{pb}{u_0}\to 0$):

- with the sinusoidal law of the change in circulation of the vortex $(\Gamma_{+i}=u_0 b\Gamma_i \sin pt)$

$$W_{i_j}=\frac{u_0\Gamma_i}{4\pi}\left(w_{i_j}\sin pt+q\frac{\partial w_{ij}}{\partial q}\cos pt\right); \tag{6}$$

- with a change in the circulation according to the cosine law $(\Gamma_{+i}(t)=u_0 b\Gamma_i \cos pt)$

$$W_{i_j}=\frac{u_0\Gamma_i}{4\pi}\left(-q\frac{\partial w_{ij}}{\partial q}\sin pt+w_{i_j}\cos pt\right). \tag{7}$$

With the harmonic law of the change in kinematic parameters of motion of the complex

$$\varphi(t)=\varphi^*\sin p_1 t,\quad h(t)=h^*\sin p_2 t,\quad \vartheta(t)=\vartheta^*\sin p_3 t$$

the increase of the coordinates of the check point in the coordinate system connected with the horseshoe-shaped vortex of the wing reflected relative to the reference surface have the form

$$\left.\begin{aligned}\Delta\eta_{i_j}=\{&\varphi^*[2\xi_j+\xi_i-\xi_{\kappa_i}-(\eta_j+\eta_i+2h_0)\operatorname{tg}\alpha_i\cos\bar\varphi_i+(\zeta_j-\zeta_i)\operatorname{tg}\alpha_i\sin\bar\varphi_i]\cos\bar\varphi_i\sin p_1 t+\\&+2h^*\cos\bar\varphi_i\sin p_2 t-2\vartheta^*[\zeta_j\cos\bar\varphi_i+(\eta_j+h_0)\sin\bar\varphi_i]\sin p_3 t\}\cos\alpha_i,\\ \Delta\zeta_{i_j}=&\varphi^*(\zeta_j+\zeta_i)\sin\bar\varphi_i\sin p_1 t+2h^*\sin\bar\varphi_i\sin p_2 t-\\&-2\vartheta^*[\zeta_j\sin\bar\varphi_i-(\eta_j+h_0)\cos\bar\varphi_i]\sin p_3 t.\end{aligned}\right\} \tag{8}$$

the increase in the coordinates of the check point in the coordinate system of the vortices, which are a mirror image with respect to the screen of the vortices located within the fuselage, has a simple form

$$\left.\begin{aligned}\Delta\eta^T_{i_j}&=(\varphi^*\xi_j\sin p_1 t+2h^*\sin p_2 t)\cos\bar\varphi^T_i-\vartheta^*(\zeta_j\cos\bar\varphi^T_i+\eta_j\sin\bar\varphi^T_i)\sin p_3 t,\\ \Delta\zeta^T_{i_j}&=(\varphi^*\xi_j\sin p_1 t+2h^*\sin p_2 t)\sin\bar\varphi^T_i-\vartheta^*(\zeta_j\sin\bar\varphi^T_i-\eta_j\cos\bar\varphi^T_i)\sin p_3 t.\end{aligned}\right\} \tag{9}$$

The coordinates of the check points entering into these expressions are computed in terms of the initial geometric parameters of the complex

$$\xi_j = 0{,}5 - \sum_{p=0}^{\kappa-1} \ell_p \operatorname{tg} \chi_p - \{0{,}75 + [\![A_j]\!]\} \frac{\eta_\kappa}{\pi_\kappa \cdot \prod_{p=1}^{\kappa} \eta_p} - \mu_j f_j ,$$

$$\eta_j = e + \sum_{p=0}^{\kappa-1} \ell_p \sin\psi_p + \mu_j \sin\psi_j - (\xi_{\kappa_j} - \xi_j)\cos\psi_j \operatorname{tg}\alpha_j ,$$

$$\zeta_j = \tau + \sum_{p=0}^{\kappa-1} \ell_p \cos\psi_p + \mu_j \cos\psi_j + (\xi_{\kappa_j} - \xi_j)\sin\psi_j \cdot \operatorname{tg}\alpha_j .$$

In turn

$$\xi_{\kappa_j} = 0{,}5 - \sum_{p=0}^{\kappa-1} \ell_p \operatorname{tg}\chi_p - \prod_{p=1}^{\kappa-1} \frac{1}{\eta_p} - \mu_j \left[\operatorname{tg}\chi_\kappa + \frac{2(1-\eta_\kappa)}{\lambda_\kappa(1+\eta_\kappa)} \right],$$

where A_j, μ_j and f_j are computed according to equations (4) with the replacement of i by j.

Representing the dimensionless circulations of the vortices in the form similar to the expansion of (1), using the dependences (2), (3), (5), (6), (7), (8), and (9) and satisfying at the check points the boundary condition, which, taking into account the nonlinearity according to the setting angle of attack, has the form

$$W_{n_j} = -u_0 \sin\alpha_j - u_0 \varphi^* \cos\alpha_j \cos\psi_j \sin p_1 t + p_2 \delta h^* \cos\alpha_j \cos\psi_j \cos p_2 t +$$
$$+ p_1 \delta \varphi^* (\eta_j \sin\alpha_j + \xi_j \cos\alpha_j \cos\psi_j) \cos p_1 t - p_3 \delta \vartheta^* (\zeta_j \cos\psi_j + \eta_j \sin\psi_j) \cos\alpha_j \cos p_3 t ,$$

we obtain a number of systems of linear algebraic equations for determining the intensities of the horseshoe-shaped vortices of the wing

$$
\left.
\begin{aligned}
&\sum_{i=1}^{m}\Gamma_i^{\circ} F_{ij} = -4\pi \operatorname{tg}\alpha_j,\\
&\sum_{i=1}^{m}\Gamma_i^{\varphi} F_{ij} = -4\pi(\cos\varphi_j - D_j) +\\
&+\sum_{i=1}^{m}\Gamma_i^{\circ}\{[\xi_{\kappa_i}-\xi_i-2\xi_j+(\eta_j+\eta_i+2h_0)\operatorname{tg}\alpha_i\cos\varphi_j+(\zeta_j-\zeta_i)\operatorname{tg}\alpha_i\sin\varphi_i]R_{ij}-(\xi_j+\xi_i)G_{ij}-\xi_j B_{ij}\},\\
&\sum_{i=1}^{m}\Gamma_i^{h} F_{ij} = -2\sum_{i=1}^{m}\Gamma_i^{\circ}(R_{ij}+G_{ij}+B_{ij}),\\
&\sum_{i=1}^{m}\Gamma_i^{\vartheta} \bar F_{ij} = 2\sum_{i=1}^{m}\Gamma_i^{\circ}[(R_{ij}+G_{ij}+0{,}5\cdot B_{ij})\zeta_j+(\eta_j+h_0)(\tilde R_{ij}-\tilde G_{ij})+0{,}5\eta_j S_{ij}],\\
&\sum_{i=1}^{m}\Gamma_i^{\dot\varphi} F_{ij} = 4\pi(\eta_j\operatorname{tg}\alpha_j+\xi_j\cos\varphi_j - L_j) - \sum_{i=1}^{m}\Gamma_i^{\varphi} Q_{ij},\\
&\sum_{i=1}^{m}\Gamma_i^{\dot h} F_{ij} = 4\pi(\cos\varphi_j - D_j) - \sum_{i=1}^{m}\Gamma_i^{h} Q_{ij},\\
&\sum_{i=1}^{m}\Gamma_i^{\dot\vartheta} \bar F_{ij} = -4\pi(\zeta_j\cos\varphi_j+\eta_j\sin\varphi_j) - \sum_{i=1}^{m}\Gamma_i^{\vartheta}\bar Q_{ij},\\
&\qquad j=1,2,3,\dots, m=\sum_{\kappa} n_\kappa N_\kappa,
\end{aligned}
\right\}
\quad (10)
$$

where

$$
\begin{aligned}
F_{ij} &= f_{ij}+\delta f_{ij}-\bar f_{ij}-\delta\bar f_{ij}, & \bar F_{ij} &= f_{ij}-\delta f_{ij}-\bar f_{ij}+\delta\bar f_{ij};\\
R_{ij} &= \bar r_{ij}\cos\bar\varphi_i+\delta\bar r_{ij}\cos\delta\bar\varphi_i, & \tilde R_{ij} &= \bar r_{ij}\sin\bar\varphi_i+\delta\bar r_{ij}\sin\delta\bar\varphi_i;\\
G_{ij} &= \bar\rho_{ij}\cos\bar\varphi_i+\delta\bar\rho_{ij}\cos\delta\bar\varphi_i, & \tilde G_{ij} &= \bar\rho_{ij}\sin\bar\varphi_i+\delta\bar\rho_{ij}\sin\delta\bar\varphi_i;\\
Q_{ij} &= q_{ij}+\delta q_{ij}-\bar q_{ij}-\delta\bar q_{ij}, & \bar Q_{ij} &= q_{ij}-\delta q_{ij}-\bar q_{ij}+\delta\bar q_{ij};\\
D_j &= v^{\circ}_{y_j}\cos\varphi_j - v^{\circ}_{z_j}\sin\varphi_j; & L_j &= \bar v^{\circ}_{y_j}\cos\varphi_j-\bar v^{\circ}_{z_j}\sin\varphi_j;\\
B_{ij} &= \bar b_{ij}+\delta\bar b_{ij}; & S_{ij} &= \bar s_{ij}+\delta\bar s_{ij}.
\end{aligned}
$$

In turn

$$\left.\begin{aligned}
f_{ij} &= w_y \cos(\psi_j-\psi_i)\frac{\cos(\alpha_j-\alpha_i)}{\cos\alpha_j} - w_z \sin(\psi_j-\psi_i) - \\
&- \sum_{\gamma=1}^{\infty}\left[w_y^{\gamma}\cos(\psi_j-\psi_i^{\gamma}) - w_z^{\gamma}\sin(\psi_j-\psi_i^{\gamma})\right] + \sum_{\lambda=1}^{\infty}\left[w_y^{\lambda}\cos(\psi_j-\psi_i^{\lambda}) - w_z^{\lambda}\sin(\psi_j-\psi_i^{\lambda})\right], \\
g_{ij} &= \frac{\partial w_y}{\partial q}\cos(\psi_j-\psi_i)\frac{\cos(\alpha_j-\alpha_i)}{\cos\alpha_j} - \frac{\partial w_z}{\partial q}\sin(\psi_j-\psi_i) - \\
&- \sum_{\gamma=1}^{\infty}\left[\frac{\partial w_y^{\gamma}}{\partial q}\cos(\psi_j-\psi_i^{\gamma}) - \frac{\partial w_z^{\gamma}}{\partial q}\sin(\psi_j-\psi_i^{\gamma})\right] + \sum_{\lambda=1}^{\infty}\left[\frac{\partial w_y^{\lambda}}{\partial q}\cos(\psi_j-\psi_i^{\lambda}) - \frac{\partial w_z^{\lambda}}{\partial q}\sin(\psi_j-\psi_i^{\lambda})\right], \\
\bar{b}_{ij} &= \sum_{\gamma=1}^{\infty}\left[\left(\frac{\partial \bar{w}_y^{\gamma}}{\partial \eta}\cos\bar{\varphi}_i^{\gamma} + \frac{\partial \bar{w}_y^{\gamma}}{\partial \zeta}\sin\bar{\varphi}_i^{\gamma}\right)\cos(\psi_j-\bar{\varphi}_i^{\gamma}) - \left(\frac{\partial \bar{w}_z^{\gamma}}{\partial \eta}\cos\bar{\varphi}_i^{\gamma} + \frac{\partial \bar{w}_z^{\gamma}}{\partial \zeta}\sin\bar{\varphi}_i^{\gamma}\right)\sin(\psi_j-\bar{\varphi}_i^{\gamma})\right] - \\
&- \sum_{\lambda=1}^{\infty}\left[\left(\frac{\partial \bar{w}_y^{\lambda}}{\partial \eta}\cos\bar{\varphi}_i^{\lambda} + \frac{\partial \bar{w}_y^{\lambda}}{\partial \zeta}\sin\bar{\varphi}_i^{\lambda}\right)\cos(\psi_j-\bar{\varphi}_i^{\lambda}) - \left(\frac{\partial \bar{w}_z^{\lambda}}{\partial \eta}\cos\bar{\varphi}_i^{\lambda} + \frac{\partial \bar{w}_z^{\lambda}}{\partial \zeta}\sin\bar{\varphi}_i^{\lambda}\right)\sin(\psi_j-\bar{\varphi}_i^{\lambda})\right], \\
\bar{s}_{ij} &= \sum_{\gamma=1}^{\infty}\left[\left(\frac{\partial \bar{w}_y^{\gamma}}{\partial \eta}\sin\bar{\varphi}_i^{\gamma} - \frac{\partial \bar{w}_y^{\gamma}}{\partial \zeta}\cos\bar{\varphi}_i^{\gamma}\right)\cos(\psi_j-\bar{\varphi}_i^{\gamma}) - \left(\frac{\partial \bar{w}_z^{\gamma}}{\partial \eta}\sin\bar{\varphi}_i^{\gamma} - \frac{\partial \bar{w}_z^{\gamma}}{\partial \zeta}\cos\bar{\varphi}_i^{\gamma}\right)\sin(\psi_j-\bar{\varphi}_i^{\gamma})\right] - \\
&- \sum_{\lambda=1}^{\infty}\left[\left(\frac{\partial \bar{w}_y^{\lambda}}{\partial \eta}\sin\bar{\varphi}_i^{\lambda} - \frac{\partial \bar{w}_i^{\lambda}}{\partial \zeta}\cos\bar{\varphi}_i^{\lambda}\right)\cos(\psi_j-\bar{\varphi}_i^{\lambda}) - \left(\frac{\partial \bar{w}_z^{\lambda}}{\partial \eta}\sin\varphi_i^{\lambda} - \frac{\partial \bar{w}_z^{\lambda}}{\partial \zeta}\cos\bar{\varphi}_i^{\lambda}\right)\sin(\psi_j-\bar{\varphi}_i^{\lambda})\right], \\
\bar{\tau}_{ij} &= -\frac{\partial \bar{w}_y}{\partial \eta}\cos(\psi_j-\bar{\psi}_i)\frac{\cos(\alpha_j-\bar{\alpha}_i)}{\cos\alpha_j} + \frac{\partial \bar{w}_z}{\partial \eta}\sin(\psi_j-\bar{\psi}_i), \\
\bar{p}_{ij} &= -\frac{\partial \bar{w}_y}{\partial \zeta}\cos(\psi_j-\bar{\psi}_i)\frac{\cos(\alpha_j-\bar{\alpha}_i)}{\cos\alpha_j} + \frac{\partial \bar{w}_z}{\partial \zeta}\sin(\psi_j-\bar{\psi}_i).
\end{aligned}\right\}\text{(II)}$$

We determine the quantities $\delta f_{ij}, \bar{f}_{ij}, \delta g_{ij}, \delta p_{ij}$ etc. by the same equations (11), having replaced the parameters of the ith vortex $\xi_{ij}, \eta_{ij}, \zeta_{ij}, \psi_i, \chi_i$ by parameters of the vortices $\delta\xi_{ij}, \bar{\xi}_{ij}, \delta\xi_{ij}, \delta\eta_{ij}$ and so on.

The parameters for the calculation of velocities induced by the intrinsic ith vortex of the right half of the wing, the vortex symmetric to it relative to the wing, and also vortices of the left half of the wing can be determined by coordinates of the check point and parameters of the transformed vortex in the following manner:

$$\xi_{i_j}=\bar{\xi}_{i_j}=\delta\bar{\xi}_{i_j}=\bar{\delta}\xi_{i_j}\approx\xi_i-\xi_j,\quad \chi_i=\bar{\chi}_i=-\delta\chi_i=-\delta\bar{\chi}_i,\quad \psi_i=-\bar{\psi}_i=-\delta\psi_i=\bar{\delta}\bar{\psi}_i,$$

$$\eta_{i_j}=[(\eta_j-\eta_i)\cos\psi_i-(\zeta_j-\zeta_i)\sin\psi_i]\cos\alpha_i+x,\quad \zeta_{i_j}=(\eta_j-\eta_i)\sin\psi_i+(\zeta_j-\zeta_i)\cos\psi_i,$$

$$\bar{\eta}_{i_j}=[(\eta_j+\eta_i+2h_0)\cos\psi_i+(\zeta_j-\zeta_i)\sin\psi_i]\cos\alpha_i-x,\quad \bar{\zeta}_{i_j}=-(\eta_j+\eta_i+2h_0)\sin\psi_i+(\zeta_j-\zeta_i)\cos\psi_i,$$

$$\delta\eta_{i_j}=[(\eta_j-\eta_i)\cos\psi_i+(\zeta_j+\zeta_i)\sin\psi_i]\cos\alpha_i+x,\quad \delta\zeta_{i_j}=-(\eta_j-\eta_i)\sin\psi_i+(\xi_j+\xi_i)\cos\psi_i,$$

$$\delta\bar{\eta}_{i_j}=[(\eta_j+\eta_i+2h_0)\cos\psi_i-(\zeta_j+\zeta_i)\sin\psi_i]\cos\alpha_i-x,\quad \delta\bar{\zeta}_{i_j}=(\eta_j+\eta_i+2h_0)\sin\psi_i+(\zeta_j+\zeta_i)\cos\psi_i;$$

$$x=(\xi_{k_i}-\xi_j)\sin\alpha_i\ .$$

Parameters for the calculation of velocities under the sum sign with respect to γ are determined by equations

$$\left.\begin{aligned}
&\xi^{\gamma}_{i_j}=\bar{\xi}^{\gamma}_{i_j}=\delta\xi^{\gamma}_{i_j}=\delta\bar{\xi}^{\gamma}_{i_j}=\xi_{i_j},\qquad \chi^{\gamma}_i=\bar{\chi}^{\gamma}_i=-\delta\chi^{\gamma}_i=\operatorname{arc}\sin\frac{l_i\,\mathrm{tg}\,\chi_i}{\sqrt{l_i^2\,\mathrm{tg}^2\chi_i+(l^{\gamma}_i)^2}},\\
&\psi^{\gamma}_i=-\bar{\psi}^{\gamma}_i=-\delta\psi^{\gamma}_i=\delta\bar{\psi}^{\gamma}_i=\pi-\operatorname{arc}\sin\frac{a^{\gamma}-b^{\gamma}}{2l^{\gamma}_i},\quad l^{\gamma}_i=\frac{1}{2}\sqrt{(a^{\gamma}-b^{\gamma})^2+(c^{\gamma}+d^{\gamma})^2},\\
&\eta^{\gamma}_{i_j}=\left(\eta_j-\frac{a^{\gamma}+b^{\gamma}}{2}\right)\cos\psi^{\gamma}_i-\left(\zeta_j-\frac{c^{\gamma}+d^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\quad \zeta^{\gamma}_{i_j}=\left(\zeta_j-\frac{c^{\gamma}+d^{\gamma}}{2}\right)\cos\psi^{\gamma}_i+\left(\eta_j-\frac{a^{\gamma}+b^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\\
&\bar{\eta}^{\gamma}_{i_j}=\left(\eta_i-\frac{\bar{a}^{\gamma}+\bar{b}^{\gamma}}{2}\right)\cos\psi^{\gamma}_i+\left(\zeta_j-\frac{\bar{c}^{\gamma}+\bar{d}^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\quad \bar{\zeta}^{\gamma}_{i_j}=\left(\zeta_j-\frac{\bar{c}^{\gamma}+\bar{d}^{\gamma}}{2}\right)\cos\psi^{\gamma}_i-\left(\eta_j-\frac{\bar{a}^{\gamma}+\bar{b}^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\\
&\delta\eta^{\gamma}_{i_j}=\left(\eta_j-\frac{a^{\gamma}+b^{\gamma}}{2}\right)\cos\psi^{\gamma}_i+\left(\zeta_j+\frac{c^{\gamma}+d^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\quad \delta\zeta^{\gamma}_{i_j}=\left(\zeta_j+\frac{c^{\gamma}+d^{\gamma}}{2}\right)\cos\psi^{\gamma}_i-\left(\eta_j-\frac{a^{\gamma}+b^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,\\
&\delta\bar{\eta}^{\gamma}_{i_j}=\left(\eta_j-\frac{\bar{a}^{\gamma}+\bar{b}^{\gamma}}{2}\right)\cos\psi^{\gamma}_i-\left(\zeta_j+\frac{\bar{c}^{\gamma}+\bar{d}^{\delta}}{2}\right)\sin\psi^{\gamma}_i,\quad \delta\bar{\zeta}^{\gamma}_{i_j}=\left(\zeta_j+\frac{\bar{c}^{\gamma}+\bar{d}^{\gamma}}{2}\right)\cos\psi^{\gamma}_i+\left(\eta_j-\frac{\bar{a}^{\gamma}+\bar{b}^{\gamma}}{2}\right)\sin\psi^{\gamma}_i,
\end{aligned}\right\}\quad (12)$$

where

$$\left.\begin{aligned}
&a^{\gamma+1}=\frac{\bar{a}^{\gamma}R^2}{(\bar{a}^{\gamma})^2+(\bar{c}^{\gamma})^2},\quad b^{\gamma+1}=\frac{\bar{b}^{\gamma}R^2}{(\bar{b}^{\gamma})^2+(\bar{d}^{\gamma})^2},\quad c^{\gamma+1}=\frac{\bar{c}^{\gamma}R^2}{(\bar{a}^{\gamma})^2+(\bar{c}^{\gamma})^2},\quad d^{\gamma+1}=\frac{\bar{d}^{\gamma}R^2}{(\bar{b}^{\gamma})^2+(\bar{d}^{\gamma})^2},\\
&\bar{a}^{\gamma}=-(2h_0+a^{\gamma}),\quad \bar{c}^{\gamma}=c^{\gamma},\quad \bar{b}^{\gamma}=-(2h_0+b^{\gamma}),\quad \bar{d}^{\gamma}=d^{\gamma}.
\end{aligned}\right\}\quad (13)$$

We compute the quantities a', b', c', and d', which correspond to the first approximation ($\gamma = 1$), according to equations (13), having assumed here that

$$\bar{a}^{0}=\eta_i+l_i\sin\psi_i,\quad \bar{b}^{0}=\eta_i-l_i\sin\psi_i,\quad \bar{c}^{0}=\zeta_i+l_i\cos\psi_i,\quad \bar{d}^{0}=\zeta_i-l_i\cos\psi_i\ :$$

We compute the parameters for the calculation of velocities

under the sum sign with respect to λ according to the same equations (12), having assumed in the first approximation ($\lambda = 1$) that:

$$\bar{a}^{\circ} = -(2h_{\circ}+\eta_i) - l_i \sin\varphi_i, \quad \bar{b}^{\circ} = -(2h_{\circ}+\eta_i) + l_i \sin\varphi_i, \quad \bar{c}^{\circ} = \zeta_i + l_i \cos\varphi_i, \; \bar{d}^{\circ} = \zeta_i - l_i \cos\varphi_i .$$

We compute the velocities w_y, w_z, and derivatives of velocities according to the Strouhal number according to equations of work [4], producing preliminarily a demeasuring by means of the root chord of the wing; derivatives of velocities $\frac{\partial w_y}{\partial \eta}, \frac{\partial w_y}{\partial \zeta}, \frac{\partial w_z}{\partial \eta}$ and $\partial w_z / \partial \zeta$ can be obtained by means of differentiating with respect to η and ζ of expressions for the appropriate velocities.

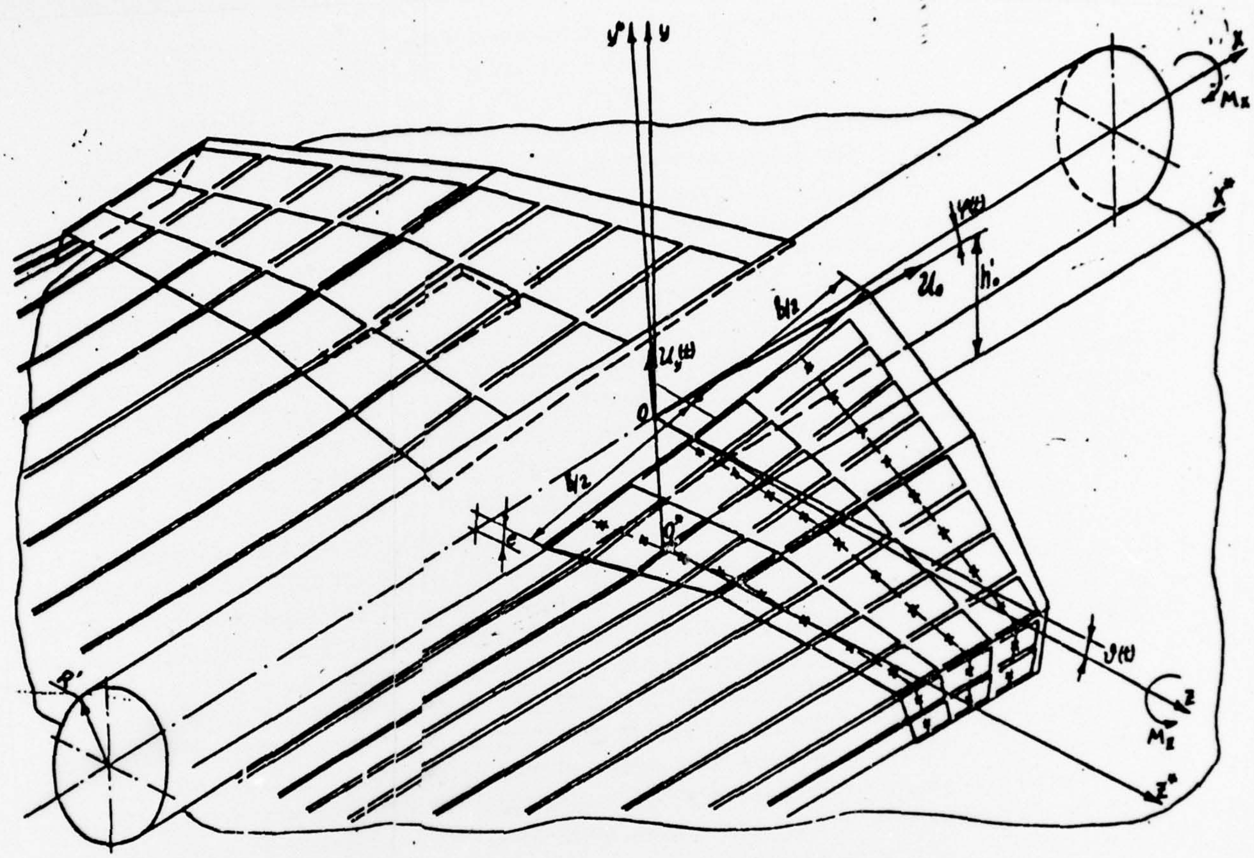
After solving the system of algebraic equations (10), it is possible, by using the Zhukovskiy formulas, to determine the total aerodynamic characteristics of the lifting wing with a cylindrical fuselage:

$$c_y^{\nu} = \frac{2Y^{\nu}}{\rho u_{\circ}^2 S} = \frac{8b^2}{S} \sum_{i=1}^{m} \Gamma_i^{\nu} l_i \cos\varphi_i, \quad m_z^{\nu} = \frac{2M_z^{\nu}}{\rho u_{\circ}^2 Sb} = \frac{8b^2}{S} \sum_{i=1}^{m} \Gamma_i^{\nu} l_i \xi_i \cos\varphi_i,$$

$$m_x^{\nu} = \frac{2M_x^{\nu}}{\rho u_{\circ}^2 Sb} = -\frac{8b^2}{S} \sum_{i=1}^{m} \Gamma_i^{\nu} l_i (\eta_i \sin\varphi_i + \zeta_i \cos\varphi_i),$$

where S is the area of the projection of the cantilevers onto the plane XOZ determined by equation

$$S = \frac{b^2}{2} \sum_{k} \left[\lambda_k (1+\eta_k)^2 \cos\varphi_k \prod_{k=1}^{k} \frac{1}{\eta_k^2} \right] .$$



Division of the wing into plates and plans and the location of horseshoe-shaped vortices on them.

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